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The Effects of Transverse Gradient of Static Magnetic Wigglers on the Free Electron Laser

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CONTENTS

THE EFFECTS OF TRANSVERSE GRADIENT OF STATIC MAGNETIC WIGGLERS ON THE FREE ELECTRON LASER

I. Introduction

The operation of the free electron laser (FEL) requires the interaction of a relativistic electron beam with the electromagnetic field combined with the wiggler field. Most of the FEL experiments in progress utilize periodic static magnetic field, $\underset{\sim}{B}_w$, which can be either linearly polarized or circularly polarized (helical magnetic field). The unrealizable one-dimensional model of the wiggler field is adequate for the study of the fundamental phenomenons of the FEL process. For more refined calculations, the realizable wiggler field, which satisfies $\nabla \cdot \underset{\sim}{B}_w = 0$ and $\nabla \times \underset{\sim}{B}_w = 0$, will have to be considered. The realizable wiggler has transverse variation as well as a non-zero axial component of the magnetic field off the axis.

In this paper, we derive the approximate equations of motion in a realizable circularly polarized wiggler field.[1,2] Those particle trajectories are used to obtain effects due to transverse gradients. We will compare the results against that due to a realizable linearly polarized wiggler field derived elsewhere[3,4]. The axial component of the magnetic field causes the betatron oscillations. The betatron oscillations of an electron beam possessing finite emittance results in an effective axial velocity spread in the axial direction for an originally cold electron beam possessing finite radius. We find that the requirement for the power density of input radiation field to initially trapping all the electrons with a circularly polarized wiggler is one quarter of the necessary input radiation field for an equivalent linearly polarized wiggler.

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II. Particle Trajectories for a Helical Wiggler

The circularly polarized, static, magnetic field of a contoured wiggler can be written as the gradient of a scalar potential

$$\Phi(r,\theta,z) \simeq \frac{-2B_w}{k_w} I_1(k_w r) \cos(k_w z - \theta), \tag{1}$$

where $\underset{\sim}{B}_w = -\nabla\Phi$, I_1 is the modified Bessel function of the first order, B_w is the amplitude of the magnetic field on axis and k_w is the wavelength of the magnetic field on axis, i.e., $r = 0$. The corresponding magnetic field is

$$\begin{aligned} \underset{\sim}{B}_w \simeq\ & B_w\left[I_o(k_w r)\cos(k_w z) + I_2(k_w r)\cos(k_w z - 2\theta)\right]\hat{e}_x \\ & + B_w\left[I_o(k_w r)\sin(k_w z) - I_2(k_w r)\sin(k_w z - 2\theta)\right]\hat{e}_y \\ & - 2\,B_w\, I_1(k_w r)\sin(k_w z - \theta)\,\hat{e}_z . \end{aligned} \tag{2}$$

For the calculation of particle trajectories, it is more convenient to write the magnetic field in terms of the vector potential $\underset{\sim}{B}_w = \nabla \times \underset{\sim}{A}_w$, where

$$\begin{aligned} \underset{\sim}{A}_w \simeq & + A_w\left[-I_o(k_w r)\cos(k_w z) + I_2(k_w r)\cos(k_w z - 2\theta)\right]\hat{e}_x \\ & - A_w\left[I_o(k_w r)\sin(k_w z) + I_2(k_w r)\sin(k_w z - 2\theta)\right]\hat{e}_y \end{aligned} \tag{3}$$

and $A_w = B_w/k_w$.

We will solve the equations of motions by making a perturbation expansion of the transverse momentum,

$$P_x = P_x^{(0)} + \delta P_x, \tag{4a}$$

$$P_y = P_y^{(0)} + \delta P_y, \tag{4b}$$

where $P_x^{(0)} >> \delta P_x$ and $P_y^{(0)} >> \delta P_y$. Furthermore we will assume the electron beam to be confined sufficiently close to the center of the axis, i.e., $1 >> k_w r$ and assume $v_z >> v_x, v_y$. Since $B_x >> B_z$ and $B_y >> B_z$, we will define

$$\frac{dP_x^{(0)}}{dt} = \frac{|e|}{c} v_{zo} B_y, \tag{5a}$$

and

$$\frac{dP_y^{(0)}}{dt} = \frac{-|e|}{c} v_{zo} B_x, \tag{5b}$$

where $v_{zo} = c\left[\frac{\gamma_o^2 - 1}{\gamma_o^2} - \beta_{o\perp}^2\right]^{1/2}$ is the axial velocity for the particle that remains on the axis and $\beta_{o\perp} = \left(\frac{|e|}{\gamma_o m_o c^2}\right)\left(\frac{B_w}{k_w}\right)$ is the transverse velocity. We will consider the steady state limit, where $\frac{\partial}{\partial t} = 0$. Making use of the fact that $v_{zo}\frac{\partial}{\partial z} >> v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y}$, we can write $\frac{d}{dt} \simeq v_{zo}\frac{\partial}{\partial z}$. To the lowest order, we find

$$P_x^{(0)} \simeq \frac{|e|}{c}\left(\underset{\sim}{A}_w(\tilde{r}, \tilde{\theta}, z) \cdot \hat{e}_x\right), \tag{6a}$$

and

$$P_y^{(0)} \simeq \frac{|e|}{c}\left(\underset{\sim}{A}_w(\tilde{r}, \tilde{\theta}, z) \cdot \hat{e}_y\right), \tag{6b}$$

where $\tilde{r}$, $\tilde{\theta}$ are functions of the Lagrangian variables (x_o, y_o, v_{xo}, v_{yo}, z), x_o, y_o are the initial transverse coordinates at z = 0, and v_{xo}, v_{yo} are the initial transverse velocities at z = 0. In the limit where $\tilde{r}$ = 0, we recover the one-dimensional result.

The equations that govern the next order corrections are:

$$\frac{d\delta P_x}{dt} = \frac{-|e|}{c} (v_{yo} B_z - B_y \delta v_z), \tag{7a}$$

$$\frac{d\delta P_y}{dt} = \frac{-|e|}{c} (\delta v_z \ B_x - v_{xo} B_z), \tag{7b}$$

where $\delta v_z = \dfrac{-(k_w r)^2}{8} v_{zo} \ \beta_{o\perp}{}^2[2-\cos 2(k_w z - \theta)]$. At this point, we perform a transformation[6-7] from the Eulerian independent variables to the Lagrangian independent variables (x_o, y_o, v_{xo}, v_{yo}, z). The total derivatives in time for the left-hand-side of Eqs. (7a,b) can be rewritten as

$$\frac{d\delta P_x}{dt} = \gamma_o m_o v_{zo} \left[v_{zo} \frac{d^2\tilde{x}}{dz^2} + \frac{d\tilde{v}_z}{dz} \frac{d\tilde{x}}{dz}\right], \tag{8a}$$

$$\frac{d\delta P_y}{dt} = \gamma_o m_o v_{zo}\left[v_{zo} \frac{d^2\tilde{y}}{dz^2} + \frac{d\tilde{v}_z}{dz} \frac{d\tilde{y}}{dz}\right], \tag{8b}$$

where

$$\frac{d\tilde{v}_z}{dz} = 2k_w \ c \ \beta_{o\perp}{}^2 I_2(k_w\tilde{r}) \ I_o(k_w\tilde{r}) \sin 2(k_w z - \theta).$$

We look for the slowly varying solutions, where $\frac{d\tilde{x}}{dz} \ll k_w\tilde{x}$ and $\frac{d\tilde{y}}{dz} \ll k_w\tilde{y}$ for $k_w\tilde{r} \ll 1$. In the slowly varying limit, it is a good approximation to assume that the expressions, which contain sinusoidal terms of ($k_w z$), contribute little to $\tilde{r}$ and $\tilde{\theta}$. The appropriate equations for $\tilde{x}$ and $\tilde{y}$ become

$$\frac{d^2\tilde{x}}{dz^2} = \beta_{o\perp}^{\ 2} k_w^{\ 2} I_1(k_w\tilde{r})[I_o(k_w\tilde{r}) \cos\tilde{\theta} - I_2(k_w\tilde{r})], \tag{9a}$$

$$\frac{d^2\tilde{y}}{dz^2} = \beta_{o\perp}^{\ 2} k_w^{\ 2} I_1(k_w\tilde{r}) I_o(k_w\tilde{r}) \sin\tilde{\theta}. \tag{9b}$$

Taylor expanding the Bessel functions and keeping only the lowest order term, Eqs. (9a,b) reduce to

$$\frac{d^2\tilde{x}}{dz^2} = K_o^{\ 2}\tilde{x}, \tag{10a}$$

$$\frac{d^2\tilde{y}}{dz^2} = K_o^{\ 2}\tilde{y}, \tag{10b}$$

where $K_o = \beta_{o\perp}k_w/\sqrt{2}$ is the wavenumber of the betatron oscillation. We have used the fact $\tilde{x} = \tilde{r}\cos\tilde{\theta}$ and $\tilde{y} = \tilde{r}\sin\tilde{\theta}$. The initial conditions are $\tilde{x}(z = 0) = x_o$, $\tilde{y}(z = 0) = y_o$, $\frac{d\tilde{x}}{dz}\big|_{z=o} = \frac{v_{xo}}{v_{zo}}$ and $\frac{d\tilde{y}}{dz}\big|_{z=o} = \frac{v_{yo}}{v_{zo}}$.

We find that electrons execute betatron oscillations in realizable wigglers with transverse gradients. The solutions to Eqs. (10a,b) are simply

$$\begin{bmatrix} \tilde{x} \\ \tilde{v}_x \end{bmatrix} = \begin{bmatrix} \cos K_o z & \frac{1}{K_o c}\sin K_o z \\ -K_o c \sin K_o z & \cos K_o z \end{bmatrix} \begin{bmatrix} x_o \\ v_{xo} \end{bmatrix}, \tag{11a, 11b}$$

and

$$\begin{bmatrix} \tilde{y} \\ \tilde{v}_y \end{bmatrix} = \begin{bmatrix} \cos K_o z & \frac{1}{K_o c}\sin K_o z \\ -K_o c \sin K_o z & \cos K_o z \end{bmatrix} \begin{bmatrix} y_o \\ v_{yo} \end{bmatrix}. \tag{12a, 12b}$$

An informative alternate approach to write Eqs. (11a) and (12a) is

$$\tilde{x} = x_B \cos(K_o z - \Phi_x), \tag{13a}$$

$$\tilde{y} = y_B \cos(K_o z - \Phi_y), \tag{13b}$$

where $x_B = [(\frac{1}{K_o} \frac{v_{xo}}{c})^2 + x_o^{\ 2}]^{1/2}$ and $y_B = [(\frac{1}{K_o} \frac{v_{yo}}{c})^2 + y_o^{\ 2}]^{1/2}$ are the amplitudes of the betatron oscillations, and $\Phi_x = \cos^{-1} (\frac{x_o}{x_B})$ and $\Phi_y = \cos^{-1} (\frac{y_o}{y_B})$ are the initial phases of the betatron oscillations in the x and y direction respectively. The transverse velocities associated with the betatron oscillations are

$$\hat{v}_x = -K_o c\ x_B \sin (K_o z - \Phi_x), \tag{14a}$$

$$\tilde{v}_y = -K_o c\ y_B \sin(K_o z - \Phi_y). \tag{14b}$$

To summarize, the particle trajectories are

$$P_x \simeq \frac{|e|}{c} (\underset{\sim}{A}_w (\tilde{r}, \tilde{\theta}, z) \cdot \hat{e}_x) + \gamma_o m_o \tilde{v}_x, \tag{15a}$$

and

$$P_y \simeq \frac{|e|}{c} (\underset{\sim}{A}_w (\tilde{r}, \tilde{\theta}, z) \cdot \hat{e}_y) + \gamma_o m_o \tilde{v}_y. \tag{15b}$$

III. Emittance and Axial Velocity Spread

All realizable electron beams have finite emittance. The emittance in the x-direction, ε_x, is defined as $\varepsilon_x = \pi\, x_{o,max}(v_{ro,max}/c)$ where $x_{o,max}$ and $v_{xo,max}$ are the electron beam waist and the maximum velocity spread respectively in the x-direction at the entrance of the magnetic wiggler. The condition for the envelope of the electron beam to remain approximately constant inside the realizable wiggler with transverse gradient is to require that particle ($x_{o,max}$, $v_{xo} = 0$) and particle ($x_o = 0$, $v_{xo,max}$) have identical betatron amplitude, i.e.,

$$x_{o,max} = v_{xo,max}/(K_o c) = (\varepsilon_x/(\pi K_o))^{1/2}. \tag{16}$$

The condition on $x_{o,max}$ in (16) also leads to the smallest electron beam radius inside the wiggler field,

$$x_b = \left(\frac{2\varepsilon_x}{\pi K_o}\right)^{1/2}. \tag{17}$$

Similarly, we will require

$$y_b = \left(\frac{2\varepsilon_y}{\pi K_o}\right)^{1/2}, \tag{18}$$

where $\varepsilon_y = \pi y_{o,max}(v_{yo,max}/c)$ is the emittance in the y-direction, $y_{o,max}$ and $v_{yo,max}$ are the electron beam waist and the maximum velocity spread respectively in the y-direction.

The gradient in the wiggler field will lead to an increase in the energy spread associated with the axial motion. Let us consider a cold electron beam with total energy $(\gamma_o - 1)\, m_o c^2$. Since part of the energy is associated with the

transverse motion, the axial velocity of a particle decreases as the amplitude of the betatron oscillation increases. Finite emittance in the transverse direction contributes to effective energy spread. The axial velocity can be written as

$$\left(\frac{\tilde{v}_z}{c}\right)^2 \simeq \frac{\gamma_o^{\,2}-1}{\gamma_o^{\,2}} - \frac{(P_x^{(0)} + \delta P_x)^2 + (P_y^{(0)} + \delta P_y)^2}{(\gamma_o m_o c)^2}.$$

We are interested in the mean axial velocity. Thus, we will drop terms that are sinusoidal functions of $k_w z$. The effective axial velocity is found to be

$$\langle \tilde{v}_z (x_o, y_o, v_{xo}, v_{yo}) \rangle = v_{zo} - \Delta v_\beta\ (x_o, y_o, v_{xo}, v_{yo}), \tag{19}$$

where $v_{zo} = c\left[\frac{\gamma_o^{\,2} - 1}{\gamma_o^{\,2}} - \beta_{o\perp}^{\,2}\right]^{1/2}$ is the axial velocity in the absence of the betatron oscillations in a non-realizable wiggler field without transverse gradient, and is also the axial velocity of the electron travelling on axis in a realizable wiggler field, $\langle(\ )\rangle = \frac{1}{L}\int_o^L(\ )dz$ is an average over axial distance L for $L \gg \frac{2\pi}{k_w}$, and Δv_β is the change in the axial velocity for a particle executing betatron oscillation. Since the axial velocity spread is proportional to the radial excursion, the maximum axial velocity spread, due to betatron oscillation alone, is

$$\Delta v_\beta = 2c(\beta_{o\perp} k_w r_b/2)^2 = (\sqrt{2}/\pi)c\ \beta_{o\perp} k_w \varepsilon, \tag{20}$$

where $r_b = x_b = y_b$ is the radius of the electron beam and $\varepsilon = \varepsilon_x = \varepsilon_y$ is the emittance for an axially symmetric beam. The corresponding longitudinal energy spread is related to the velocity spread by $\Delta E_\beta = \gamma_{zo}^{\,2}(\Delta v_\beta/c)\gamma_o m_o c^2$. The longitudinal energy spread in terms of the radius of the electron beam and the

emittance are spectively

$$\frac{\Delta E_\beta}{\gamma_o m_o c^2} = 2\left(\frac{\gamma_{zo}\beta_{o\perp}k_w r_b}{2}\right)^2 = (\sqrt{2}/\pi)\ \gamma_{zo}{}^2\beta_{o\perp}k_w\varepsilon. \tag{21}$$

One efficiency enhancement approach is to initially trap a large fraction of the electrons in the ponderomotive potential well and adiabatically extract kinetic energy from the particles. In order to trap a substantial fraction of the electrons, we require the trapping potential to be larger or at least comparable to the axial energy spread, i.e., $|e|\phi_{trap} > \Delta E_\beta$. The initial depth of the trapping potential is

$$|e|\phi_{trap}/(\gamma_o m_o c^2) = 4\gamma_{oz}\beta_{o\perp}(A_R/A_w)^{1/2}. \tag{22}$$

Combining Eqs. (20), (21) and (22), we obtain the requirement for the input radiation field in terms of the radius of the electron beam,

$$A_R > (\gamma_{zo}\ \beta_{o\perp}\ k_w{}^2 r_b{}^2)^2\ \frac{A_w}{64}. \tag{23}$$

Since the optimal radius is determined by the emittance, we can also rewrite the input radiation field, required for trapping the electrons, in terms of emittance

$$A_{R,Cir} > \left(\frac{\gamma_{zo}\ k_w\ \varepsilon}{\pi}\right)^2 \frac{A_{w,Cir}}{8}. \tag{24}$$

Now, we will compare the effects of transverse gradient on identical and axially symmetric electron beams. When the transverse gradient is not considered, we have previously found that the FEL action utilizing a linearly polarized wiggler[6] and a circularly polarized wiggler are qualitatively

equivalent if $A_{w,Cir} = A_{w,Lin}/\sqrt{2}$ and $A_{R,Cir} = A_{R,Lin}/\sqrt{2}$. Using these relations, we notice that

$$K_{o,Cir} = K_{o,Lin}/\sqrt{2},$$

$$r_{b,Cir} = 2^{1/4}\, r_{b,Lin}, \tag{25}$$

$$\Delta\gamma_{\beta,Cir} = \Delta\gamma_{\beta,Lin}/\sqrt{2},$$

and

$$\phi_{trap,Cir} = \phi_{trap,Lin}/\sqrt{2}.$$

The wavelength of the betatron oscillation is longer; the minimum radius of the electron beam in the wiggler is larger and the axial velocity spread is smaller for the circularly polarized wiggler than for the linearly polarized wiggler. The mimimum radiation field required for trapping the electrons for the linearly polarized wiggler is

$$A_{R,Lin} > \left(\frac{\gamma_{zo}\, k_w\, \varepsilon}{\pi}\right)^2 \frac{A_{w,Lin}}{4}. \tag{26}$$

The ratio of the input radiation power densities required for trapping is

$$P_{Cir} = \frac{1}{4}\, P_{Lin}, \tag{27}$$

where $P_{Cir} = (4\pi c)^{-1}\, \omega^2 A^2_{R,Cir}$ and $P_{Lin} = (8\pi c)^{-1}\, \omega^2 A^2_{R,Lin}$. The minimum initial power density required for trapping the electrons for a circularly polarized wiggler is only one quarter of that for a linearly polarized wiggler. Therefore, if the emittance combined with the betatron oscillation is the largest source of energy spread, then the circularly polarized wiggler is a better choice for axially symmetric electron beams.

IV. Numerical Results

We illustrate the effect of transverse gradients within a realizable magnetic wiggler with the following numerical example. We consider a wiggler with a period $\ell_w = 2.8$ cm ($k_w = 2.24$ cm^{-1}), and magnetic field $B_{w,Lin} = 5$ kG ($A_{w,Lin} = 2.23 \times 10^3$ statvolts) for a linearly polarized wiggler or $B_{w,Cir} = 5\sqrt{2}$ kG ($A_{w,Lin} = 1.58 \times 10^3$ statvolts) for a circularly polarized wiggler; and we consider an electron beam with energy of 25 MeV ($\gamma_o = 50$) and $\varepsilon = 2\pi$ mm mr. The parameters are appropriate for radiation at 10.6 μm. The wavelength of the betatron oscillation is 76 wiggler periods ($K_{o,Cir} = 2.9 \times 10^{-2}$ cm^{-1}) for the circularly polarized wiggler and is 54 wiggler periods ($K_{o,Lin} = 4.1 \times 10^{-2}$ cm^{-1}) for the linearly polarized wiggler. We find that the optimal radii of the electron beam within the wiggler field are $r_{b,Cir} = 0.12$ cm and $r_{b,Lin} = 0.1$ cm. These satisfy our approximation $k_w r_b \ll 1$. The input radiation power densities required for trapping are $P_{Cir} = 25$ MW/cm^2 and $P_{Lin} = 97$ MW/cm^2.

First, we will examine the accuracy of the expression for the particle trajectories, Eqs. (15a, b). We will define

$$\tilde{\sigma}_x = \left[P_{x,exact} - \left(\frac{|e|}{c}\left(\underset{\sim}{A}_w(\tilde{r}, \tilde{\theta}, z) \cdot \hat{e}_x\right) + \gamma m_o \tilde{v}_x\right)\right]/P_{\perp o}, \qquad (28a)$$

and

$$\tilde{\sigma}_y = \left[P_{y,exact} - \left(\frac{|e|}{c}\left(\underset{\sim}{A}_w(\tilde{r}, \tilde{\theta}, z) \cdot \hat{e}_y\right) + \gamma m_o \tilde{v}_y\right)\right]/P_{\perp o}, \qquad (28b)$$

where $P_{x,exact}$ and $P_{y,exact}$ are particle momentums in the x and y-directions obtained numerically using the exact wiggler magnetic fields Eq. (2), and $P_{\perp o}$ is the total initial transverse momentum. Figure 1 shows plots of average $\tilde{\sigma}_x$ and $\tilde{\sigma}_y$ as a function of axial distance z for a particle, that is initially

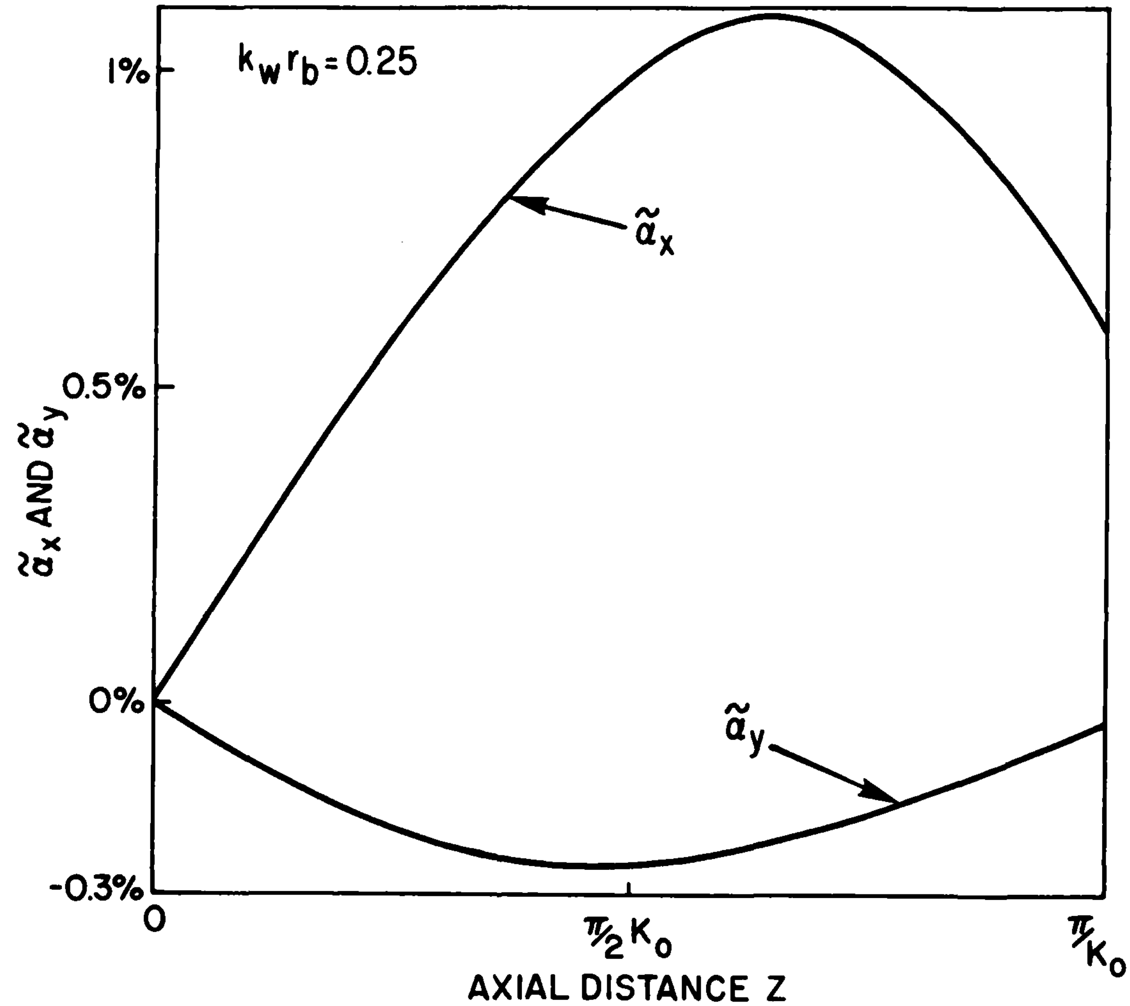

Fig. 1 — Plots of the average error of the approximate electron momentum in the transverse directions, $\tilde{\alpha}_x$ and $\tilde{\alpha}_y$, versus axial distance z associated with the particle $k_w x_o = 0.25$ and $y_o = v_{xo} = v_{yo} = 0$

at $x_o = .12$ cm and $y_o = v_{xo} = v_{yo} = 0$. For this example, $k_w r_b \simeq 0.25$, and the error is only 1%. Next, we illustrate betatron oscillations for $k_w r_b$ comparable to 1. Figure 2 contains plots of $\tilde{x}$ as a function of axial distance z for various values of $k_w r_b$ with $v_{xo} = v_{yo} = 0$. We find that as $k_w r_b$ increases the wavelength of the betatron oscillation decreases, while the amplitude stays constant. Thus, the effective axial velocity spread becomes larger than the expression shown in Eq. (20) as $k_w r_b$ becomes larger.

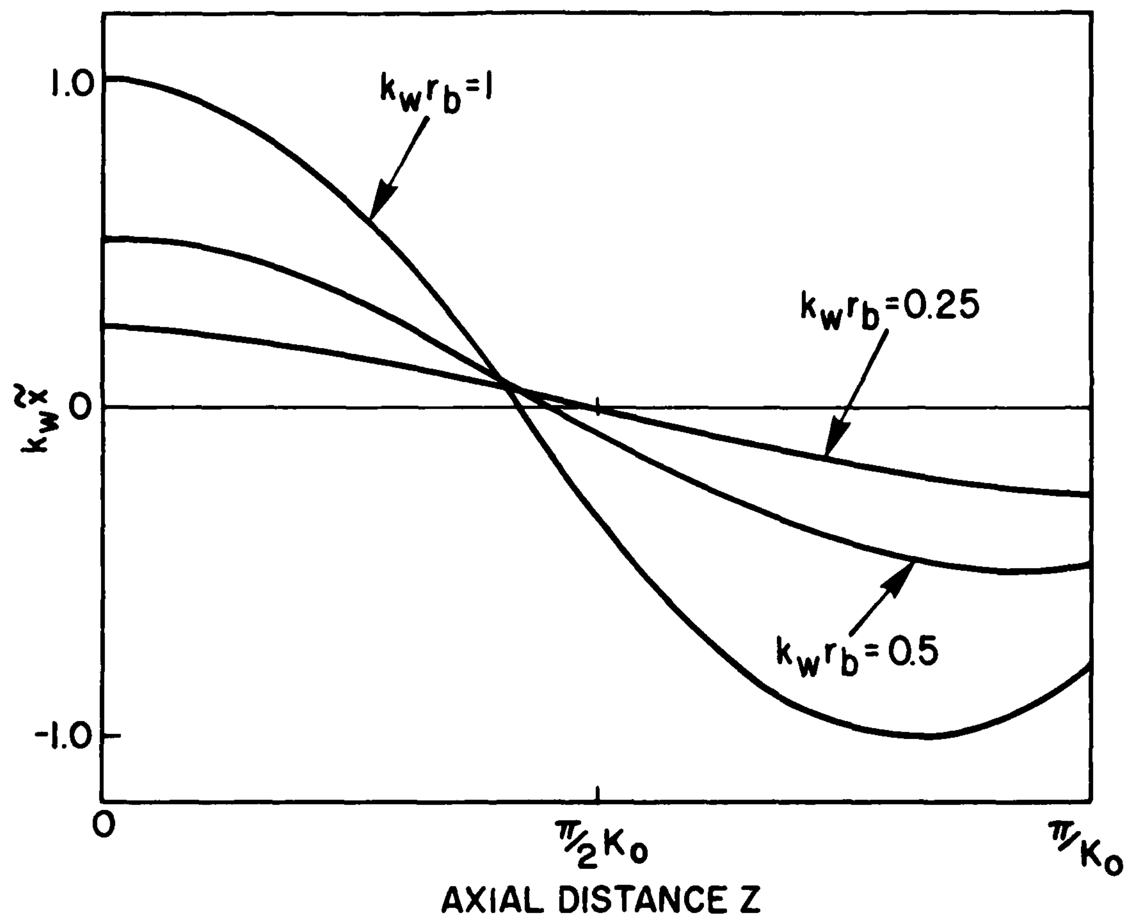

Fig. 2 — Plots of $\tilde{x}$ as a function of axial distance z for particles with $y_o = v_{xo} = v_{yo} = 0$ and various values of $k_w r_b$

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Appendix I

Betatron Oscillations Due To Transverse Gradient Of The Linearly Polarized Wiggler Field

This appendix will obtain the particle's transverse motion and the axial velocity spread inside a linearly polarized realizable wiggler field. For the illustrative purposes here, we will assume that the wiggler amplitude and wavelength are constant in the axial direction. The vector potential of the wiggler field is

$$\underset{\sim}{A}_w(y,z) = A_w \cosh(k_w y) \cos(k_w z)\, \hat{e}_x. \tag{I.1}$$

The associated wiggler magnetic field has a z component off the axis, in addition to the y component,

$$\underset{\sim}{B}_w(y,z) = -A_w k_w [\cosh(k_w y) \sin(k_w z)\, \hat{e}_y + \sinh(k_w y) \cos(k_w z)\, \hat{e}_z]. \tag{I.2}$$

For particle trajectories in the transverse directions, it is sufficient to assume that their motion is effected by the wiggler field alone, i.e., $P_x(y,z) \simeq \frac{|e|}{c} A_w$.

The particle location in the x-direction can be obtained by integrating the momentum in the x direction.

$$\tilde{x} \simeq x_o + \frac{\beta_{o\perp}}{k_w} \cosh(k_w \tilde{y}) \sin k_w z + \frac{v_{xo}}{c} z \tag{I.3}$$

where $\tilde{x}$ and $\tilde{y}$ are functions of (z, x_o, y_o, v_{yo}), x_o and y_o are the initial transverse coordinates, v_{yo} is the initial transverse velocity in the y-direction.

The particle motion in the y-direction is due to the z component of the wiggler field,

$$\frac{dP_y}{dt} = \frac{|e|}{c} v_x B_z. \tag{I.4}$$

We will assume that the fast oscillatory terms, with wavelength half the wiggler wavelength, are unimportant, and replace v_z by c at appropriate places. We find that

$$\frac{d^2\tilde{y}}{dz^2} + \frac{\beta_{o\perp}^2}{4} k_w \sinh 2k_w\tilde{y} = 0 \tag{I.5}$$

where $\beta_{o\perp} = |e|A_w/(\gamma_o m_o c^2)$. This equation can be integrated once to give

$$\frac{d\tilde{y}}{dz} = [\frac{\beta_{o\perp}^2}{4} (\cosh(2k_w y_o) - \cosh(2k_w\tilde{y})) + (\frac{v_{yo}}{c})^2]^{1/2}. \tag{I.6}$$

To find an explicit solution of y as a function of z, we expand cosh in Taylor series around zero, and keep only the first two terms of the expansion. Then, Eq. (I.6) can be rewritten as

$$z = \frac{\sqrt{2}}{\beta_{o\perp}k_w} \int_{y_o}^{\tilde{y}} [2(\frac{v_{yo}/c}{\beta_{o\perp}k_w})^2 + y_o^2 - \tilde{y}^2]^{1/2}. \tag{I.7}$$

The results of the integral can be put into the following form

$$\begin{bmatrix} \tilde{y} \\ \tilde{v}_y \end{bmatrix} = \begin{bmatrix} \cos K_o z & (K_o c)^{-1} \sin K_o z \\ -K_o c \sin K_o z & \cos K_o z \end{bmatrix} \begin{bmatrix} y_o \\ v_{yo} \end{bmatrix} \tag{I.8}$$

where $K_o = \beta_{o\perp}k_w/\sqrt{2}$ is the wavenumber of the betatron oscillations. An equally convenient form for $\tilde{y}$ is

$$\tilde{y} = y_B \cos (K_o z - \phi_B), \tag{I.9}$$

where

$$y_B = \left[2\left[\frac{v_{yo}/c}{\beta_{o\perp}k_w}\right]^2 + y_o^2\right]^{1/2} \tag{I.10}$$

is the amplitude of the betatron oscillation and

$$\phi_B = \cos^{-1}(y_o/y_B) \tag{I.11}$$

is the initial phase of the betatron oscillation.